Standard spin-$^1/_2$ quantum kicked rotor in the inhomogeneous magnetic field

1995 J. Phys. A: Math. Gen. 28 L147

(http://iopscience.iop.org/0305-4470/28/5/001)

J. Phys. A: Math. Gen. 28 (1995) L147–L150. Printed in the UK

LETTER TO THE EDITOR

Standard spin-$\frac{1}{2}$ quantum kicked rotor in the inhomogeneous magnetic field

D R Mašović

Institute of Nuclear Sciences 'Vinča', Laboratory for Theoretical Physics, PO Box 522, 11001 Beograd, Serbia, Yugoslavia

Received 1 December 1994

Abstract. The problem of anti-unitary symmetry breaking in the model of a standard spin-$\frac{1}{2}$ quantum kicked rotor in an inhomogeneous magnetic field is examined in relation to the localization properties. It is shown that the functional form of the spin-dependent term in the kicking potential (not the anti-unitary symmetries) is responsible for localization length changes. Our numerical results for the proposed functional form yield evidence of an anomalous and continuous growth of the localization length with an increase in the magnitude of the magnetic field.

Quantum mechanical periodically driven systems, usually represented by the quantum kicked rotor (QKR), have been extensively studied in recent years. It has been found that unbounded diffusion as a characteristic of deterministic chaos in classical systems was suppressed in the quantum case and this is known as the phenomenon of dynamical localization. The importance of time-reversal symmetry in QKR models with dynamical localization was especially examined in relation to the localization length changes. The various models were investigated [1, 2] and the results obtained are compared with the known estimation of the localization length for quasi-one-dimensional disordered systems [3] in accordance with the confirmed similarity between the dynamical localization of QKR and the Anderson localization of disordered systems [4].

It was generally accepted that the localization length ξ for finite QKR [2] satisfies

$$\xi(\beta) = \beta\xi(\beta = 1) \tag{1}$$

where β is equal to 1 or 4 for the time-reversal invariant QKR with or without a spin degree of freedom (symplectic case) and 2 for QKR with broken time-reversal symmetry. This is in agreement with the prediction based on the random matrix treatment of localization in quasi-one-dimensional disordered systems [3].

Result (1) suggests that the localization length of QKR is a universal function of symmetry class. Moreover, in their recent paper Thaha and Blümel [5] have shown that (1) holds only if the switching between symmetry classes is so performed that Kramers' degeneracy is preserved. Their main conclusion is that the localization length of the QKR may not be a universal function of its symmetry class. Concerning the non-universality features discussed in [5], there is a comment on this paper by Mirlin [6] in which the author explains the earlier results [5] and argues that the universality nevertheless exists.

In a previous paper [7] we considered the standard QKR model augmented by a spin-$\frac{1}{2}$ in a homogeneous magnetic field. On the basis of the mentioned analogy between the

0305-4470/95/050147+04$19.50

dynamical localization in QKR and Anderson localization of disordered systems, the QKR model is mapped on the corresponding tight-binding model. Then it is shown, examining the latter one, that a magnetic field which breaks the conventional time-reversal symmetry (and trivial Kramers' degeneracy), in agreement with [6], does not have an influence on localization. So it is reasonable to show how the localization length depends on the functional form of the spin dependent term in the kicking potential (formula (2), below), retaining the time-reversal exactly the same as in the previous case [7]. This is our primary motivation for studying the QKR model in an inhomogeneous magnetic field. A specific localization behaviour is also expected, rather than the one well known for the standard spinless QKR [8, 9].

Let us suppose that the spin-$\frac{1}{2}$ particle is in the ring located in the $x0y$ plane and that the electric and magnetic fields are along the x axis making the magnetic field inhomogeneous across the ring. Following the notation introduced in [7], the Hamiltonian has the form

$$\mathcal{H} = -\frac{\tau}{2}\frac{\partial^2}{\partial\theta^2}\mathbf{1} + k\cos\theta\sum_n \delta(t-n)\mathbf{1} + H|\cos\theta|\sigma_x\sum_n \delta(t-n) \tag{2}$$

where τ and k are the two standard parameters of a spinless QKR [4], H corresponds to the magnitude of the θ dependent magnetic field, $\mathbf{1}$ is the (2×2) unit matrix and σ_x is the Pauli spin matrix.

The Hamiltonian (2) is invariant under the non-conventional time-reversal T [7] such that we have $TUT^{-1} = U^{+}$, where U is the corresponding Floquet operator defined as in [7] and $T^2 = 1$, which means that the conventional time-reversal and Kramers' degeneracy are broken by the magnetic field.

In order to study the quantal behaviour of the model we have to solve the eigenvalue problem $Uu = \mathrm{e}^{-\mathrm{i}\omega}u$, where u denotes the spinor and ω is the quasi-energy. The behaviour of the model is completely determined by the so obtained quasi-energy states.

Moreover, in this letter we have decided to follow the approach as in [7], mapping the Floquet eigenvalue problem on the tight-binding model of Shepelyansky's type [8]. Such a procedure is the most convenient and it leads to a new Hamiltonian with a band structure as in solid-state physics. Then it is possible to exploit the band structure to study localization properties by standard means of transfer matrices [9].

Using the expansion rate

$$\exp(\mathrm{i}k\cos\theta)\exp(\mathrm{i}H|\cos\theta|) = \sum_r W_r(k, H)\exp\left[\mathrm{i}\left(r\theta + r\frac{\pi}{2}\right)\right] \tag{3}$$

we will get, by simple generalization of mapping procedure [8], the following tight-binding model:

$$\sum_{r=-b}^{b}\left[X_r\sin\left(\varphi_n - r\frac{\pi}{2}\right) \pm Y_r\cos\left(\varphi_n - r\frac{\pi}{2}\right)\right]u_{n+r} = 0 \tag{4}$$

where $\varphi_n = \frac{1}{2}\omega - \frac{1}{4}\tau n^2$ and X_r, Y_r are the real and imaginary parts of W_r^* respectively. Since the Fourier components W_r decay as a power law of r it is assumed that only X_r, Y_r with $|r| \leqslant b$ differ from zero [9]. The signs $+$ or $-$ in (4) correspond to the case when the electric and magnetic fields are in the same or opposite direction in the ring. We shall refer to these cases as $(+)$ and $(-)$ respectively.

In order to demonstrate the localization length behaviour we will take the phase factors φ_n in (4) as randomly distributed in the interval $[-\pi/2, \pi/2]$. Assuming that the kicking frequency is $\sim$ 10 GHz and the applied magnetic field is a maximum of 6 T, then the upper limit for the parameter H is $\sim$ 50. Varying H up to 16, which is the limit of our computational possibilities, we can follow the localization length changes against parameter H. The main feature of the proposed model is the anomalous growth of the localization length with an increase H. Figure 1 shows the dependence ξ/ξ_0, where ξ_0 is the localization length for the spinless case, as a function of H for $k = 8$–12 (case (+)). Since agreement between the cases (+) and (−) has been found, only the results for the case (+) are given here. The best fit gives polynomial dependence of the localization length on H:

$$\xi/\xi_0 \approx 0.016\,24H^2 - 0.075\,56H + 1.032\,09 \qquad (k = 8)$$

which clearly shows that the estimation of the localization by Shepelyanksky, $\xi \sim b^2$ [8], does not hold here.

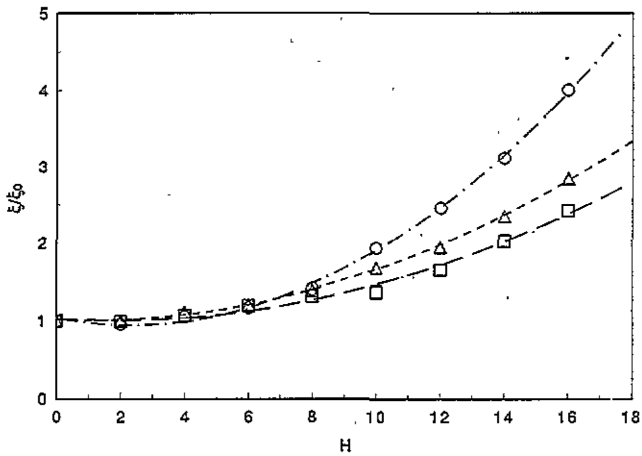

Figure 1. ξ/ξ_0 as a function of H for $k = 8$ (O), $k = 10$ (Δ) and $k = 12$ ($\Box$).

In conclusion we can say that the breaking of the conventional time-reversal by magnetic field in the standard version of spin-$\frac{1}{2}$ QKR does not have an influence on the localization length changes. The previous results [7] support this statement. The functional form of the spin-dependent term in the kicking potential is exclusively responsible for a possible difference in behaviour of quasi-energy states. With the proposed functional form in (2) we have obtained the anomalous and continuous growth of the localization length for increasing value of H ($H \gtrsim k$). All calculations indicate a polynomial of quadratic degree as the optimal approximation for the localization length dependence on H, so that the new behaviour co-exists in this model with the well known behaviour for the spinless QKR [8, 9].

References

[1] Scharf R 1989 *J. Phys. A: Math. Gen.* **22** 4223

[2] Blümel R and Smilansky U 1992 *Phys. Rev. Lett.* **69** 217

[3] Pichard J-L, Sanquer M, Slevin K and Debray P 1992 *Phys. Rev. Lett.* **65** 1812

[4] Fishman S, Grempel D R and Prange R E 1982 *Phys. Rev. Lett.* **49** 509
Grempel D R, Prange R E and Fishman S 1984 *Phys. Rev.* A **29** 1639

[5] Thaha M and Blümel R 1994 *Phys. Rev. Lett.* 72 72
[6] Mirlin A D 1994 *Phys. Rev. Lett.* 72 3437
[7] Mašović D R and Tančić A R 1994 *Phys. Lett.* **191A** 384
[8] Shepelyansky D L 1986 *Phys. Rev. Lett.* 56 677
Shepelyansky D L 1987 *Physica* **28D** 103
[9] Blümel R, Fishman S, Griniasty M and Smilansky U 1986 Quantum chaos and statistical nuclear physics *Proc. 2nd Int. Conf. on Quantum Chaos (Curneraca, Mexico)* ed T H Seligman and H Nishioka (Heidelberg: Springer) p 212